MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - March 2023
Practice Problems

1A) Solve for x: $\begin{vmatrix} 2x & 3 \\ 7 & x \end{vmatrix} = \begin{vmatrix} 0 & x & -2 \\ -1 & 0 & 1 \\ 2 & 3 & 1 \end{vmatrix}$

2A) Compute all values of x for which $\left(4^x\right)^3 = 8^{6^2}$.

3A) Compute $\operatorname{Cot}^{-1}(-1) + \operatorname{Sin}^{-1}\left(-\frac{1}{2}\right)$.

<u>Note:</u> There is uniform agreement on the range of the following principal inverse trig functions: $y = \operatorname{Sin}^{-1}(x)$, $y = \operatorname{Cos}^{-1}(x)$, and $y = \operatorname{Tan}^{-1}(x)$

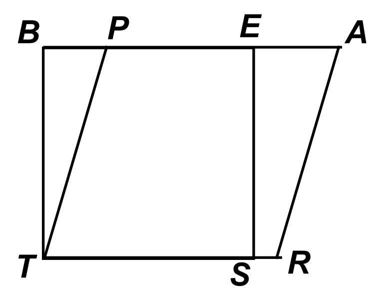
$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \qquad [0, \pi] \qquad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

There is **<u>no</u>** uniform agreement for $y = \operatorname{Cot}^{-1}(x)$, $y = \operatorname{Sec}^{-1}(x)$, and $y = \operatorname{Csc}^{-1}(x)$.

For this question, assume the range of $y = \operatorname{Cot}^{-1}(x)$ is $(0, \pi)$.

4A) A box measuring 6” wide, 8” deep, and 15” high is completely filled with 10,000 identical buttons. 6400 of these buttons are packed with the same density in a container having the same footprint (6” x 8”). Compute the number of inches in the height to which this container is filled.

5A) *BEST* is a square with side-length 48. *P* is on $\overline{BE}$ such that $BP : PE = 7 : 17$. If *PART* is a rhombus, compute *PR*.



6A) Let *S* be the sum of the coefficients of $(x-2y-3)^2$ when the expression is expanded, and any similar terms are combined. Likewise, let *T* be the sum of the coefficients of $(x+2y+3)^2$. Compute $\frac{S}{T}$.

Team D)
The region bounded by the graph of $|x| + |y| = 7$ is divided into two regions by the graph of $y = 2(1-x)$. Compute the ratio of the area of the larger of these two regions to the area of the other.

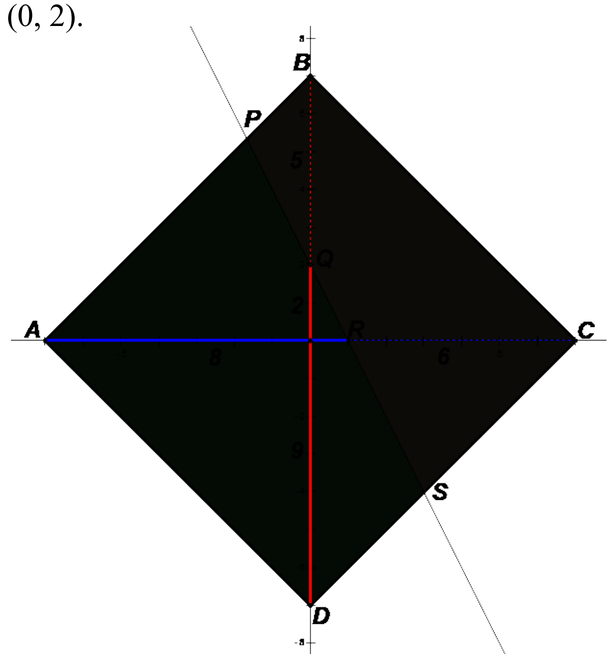
Answers:

1A)	$-3, \frac{9}{2}$	4A)	9.6
2A)	18	5A)	60
3A)	$\frac{7\pi}{12}$	6A)	$\frac{4}{9}$

Team D) $\frac{25}{17}$

The region bounded by the graph of $|x|+|y|=7$ is divided into two regions by the graph of $y=2(1-x)$. Compute the ratio of the area of the larger of these two regions to the area of the other.

The graph of $|x|+|y|=7$ is a square with vertices on the *x*- and *y*-axes.
The graph of $y=2(1-x)$ is a line with intercepts (1, 0) and (0, 2).



$\Delta BPQ \sim \Delta DSQ$ and (BQ, DQ) = (5, 9).
Let $PB = 5a$ and $SD = 9a$.
$\Delta SCR \sim \Delta PAR$ and (CR, AR) = (6, 8).
Let $SC = 3b$ and $PA = 4b$.

$$\begin{cases} AB = PA + PB = 4b + 5a = 7\sqrt{2} \\ CD = SC + SD = 3b + 9a = 7\sqrt{2} \end{cases}.$$

$$\Rightarrow b - 4a = 0 \Leftrightarrow b = 4a$$

$$\frac{\text{area}(APSD)}{\text{area}(PBCS)} = \frac{\cancel{\frac{1}{2}AD}\,(4b+9a)}{\cancel{\frac{1}{2}BC}\,(5a+3b)} = \frac{25a}{17a} = \underline{\mathbf{\frac{25}{17}}}.$$