MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - November 2022
Practice Problems

1A) Compute $\frac{2}{1+i}+(1-i)^2+\sqrt{-16}$.

2A) There are 5 children in the Robinson family. Tom is 8 years younger than Chris. Samantha is half as old as Chris. Bonnie and Haley are twins whose combined ages are equal to that of Tom. Today is not the twins' birthday. If the sum of the children's ages is 54 years, how old were the twins the day before their last birthday?

3A) *PQRS* is a square. *M* and *N* are midpoints of $\overline{PS}$ and $\overline{PQ}$, respectively.
U is the trisection point of $\overline{SR}$ closer to *S*.
V is the trisection point of $\overline{QR}$ closer to *Q*.
Compute the area of *PQRS*, if the area of hexagon *MSUVQN* is 94.

4A) Factor completely over the integers: $(25x-84)^2-x^4$

5A) Compute $\sec\theta$, if $\tan\theta=-\frac{21}{20}$ and θ is located in quadrant 2.

6A) The measure of each interior angle of a regular polygon is nine times the measure of an exterior angle. How many diagonals does this polygon have?

Team B)
$P(4,-2)$, $Q\left(x,\frac{5}{2}\right)$, and $R(-2,y)$ are collinear, and $PQ = QR$.
Let *A* and *B* be the points where $\overleftrightarrow{PQ}$ intersects the *x*- and *y*-axes, respectively.
Let *C* be the point where the line perpendicular to $\overleftrightarrow{PQ}$ at *A* intersects the *y*-axis.
Compute the area of ΔABC.

Answers:

1A)	$1+i$	4A)	$(21-x)(4-x)(3-x)(28+x)$ or equivalent
2A)	5	5A)	$-\dfrac{29}{20}$ (–1.45 or equivalent)
3A)	144	6A)	170

Team B) $\dfrac{208}{27}$

$P(4,-2)$, $Q\left(x,\frac{5}{2}\right)$, and $R(-2,y)$ are collinear, and *PQ* = *QR*.
Let *A* and *B* be the points where $\overleftrightarrow{PQ}$ intersects the *x*- and *y*-axes, respectively.
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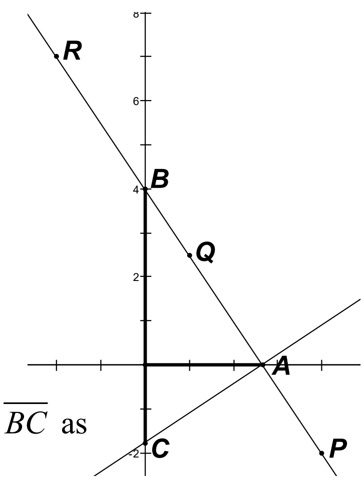
Compute the area of ΔABC.

Since Q (x, y) must be the midpoint of $\overline{PR}$, $x=\dfrac{4+(-2)}{2}=1$, $\dfrac{5}{2}=\dfrac{-2+y}{2}\Rightarrow y=7$.

Therefore, the slope of $\overleftrightarrow{PR}$ is $\dfrac{7-(-2)}{-2-4}=-\dfrac{3}{2}$, and the equation is $y+2=-\dfrac{3}{2}(x-4)$

$\Rightarrow 3x+2y=8\Rightarrow A\left(\dfrac{8}{3},0\right)$, $B(0,4)$.



Since the slope of a perpendicular to $\overleftrightarrow{PQ}$ must be $\dfrac{2}{3}$, the perpendicular through A will have the equation $(y-0)=\dfrac{2}{3}\left(x-\dfrac{8}{3}\right)\Leftrightarrow 6x-9y=16$ and the y-intercept is $C\left(0,-\dfrac{16}{9}\right)$.

$\overline{AB}\perp\overline{AC}$, but rather than computing the distances between the endpoints of those segments to find the area of ΔABC, we utilize the vertical segment $\overline{BC}$ as the base, and the altitude from A (which is horizontal).

The area of ΔABC is $\dfrac{1}{2}\left(4+\dfrac{16}{9}\right)\left(\dfrac{8}{3}-0\right)=\dfrac{1}{2}\cdot\dfrac{52}{9}\cdot\dfrac{8}{3}=\dfrac{\mathbf{208}}{\underline{\mathbf{27}}}$.