MASSACHUSETTS MATHEMATICS LEAGUE

CONTEST 1 - October 2022

Practice Problems

1A) The circumference of a great circle of a sphere is . Compute the ratio of the number of cubic units in the volume of the sphere to the number of square units in its surface area.



2A) Quadrilaterals *PATH*, *HILO*, and *ROPE* are squares.

Compute the area of *ROPE*, if and .

3A) The sum of 9 consecutive positive integers is divisible by 12.

*S* is the sum of the second and seventh numbers.

Compute the minimum value of *S*.

4A) For all values of *a*, dividing  by , where , leaves the same remainder *k*. Compute *k*.

5A) For integers *x* and *y*, and .

Compute the minimum value of .

6A) If , determine the value(s) of *x* for which is undefined.

Team F)

Determine the number of ordered pairs  for which the base-11 integer **** is divisible by 4.

**Answers:**

1A)  4A) ‒16

2A) 146 5A) 24

3A) 15 6A) ‒ 6

Team F) 30

**Determine the number of ordered pairs  for which the base-11 integer  is divisible by 4.**

In base 10,

* the place values of each digit are powers of 10,

e.g., 

* the 10 available digits are 0, 1, 2,..., 8, and 9, the largest being 1 less than the base
* divisibility by 4 depends only on the rightmost two digits, since the number formed by the other digits will be a multiple of 100, guaranteeing divisibility by 4.

In base 11,

* place values are powers of 11
* the 11 available digits are 0, 1, 2, ..., 9, 10, where 10 is usually represented by a single character (like *t*) so as not to be confused with a two-digit number
* divisibility by 4 depends on all the digits, not just the rightmost two.

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⇒ 

Thus, we must ensure that is a multiple of 4, where , and .



There are **30** ordered pairs for which divisibility by 4 is guaranteed.