MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 4 - January 2022 Practice Problems

- 1A) Find the equation of the line through (-2, 4) tangent to the circle with the equation $(x+3)^2 + (y-6)^2 = 5$. Express your answer in Ax + By = C form, where A, B and C are integers and A > 0.
- 2A) For integers A, B, C, and D, (5x+2)(6x-5)-26 = (Ax+B)(Cx+D), where A > C > 0. Determine the ordered quadruple (A, B, C, D).

В

F

G

B

С

A

D

- 3A) For certain integer values of k, x = A satisfies the equation $2\cos^2 x = k\sin x + 1$. If A is a quadrantal value in the range $0 \le x < 2\pi$, compute all ordered pairs (k, A).
- 4A) Given: x(5x+4) = 3(2-3x)Let *D* be the absolute value of the difference of the roots of the equation. Let *Q* denote the larger quotient of the roots. Compute the ordered pair (D,Q).
- 5A) Given: $\overline{DE} \parallel \overline{BC}$, AD = 5, DB = kIf the area of $\triangle ADE$ is 144, and the area of *DECB* is 297, compute *k*.
- 6A) At the start of 2021, there was a total of \$320 in Louie's bank account. For *n* subsequent weeks, his take-home pay, which was deposited into his bank account, increased by \$7 per week; unfortunately, his weekly expenses increased by \$12. At the end of the *n* weeks, his bank account had lost $62\frac{1}{2}$ % of its value. Then his luck changed. For the remaining weeks of the year, he was able to deposit enough money each week to build his bank account up to \$432. How much did he deposit each week? (Assume there are 52 weeks in a year.)

Team E)

Given: $\overline{GH} \perp \overline{BD}$, DJ = 7, JI = 24, GH = 9.8 *ABCD* is a rectangle and *AFGE* is a square. Compute the ratio of the area of trapezoid *GIDE* to the area of trapezoid *BIJC*.

Answers:

- 1A) x 2y = -10 4A) $\left(\frac{17}{5}, -\frac{2}{15}\right)$
- 2A) (10,9,3,-4) 5A) $\frac{15}{4}$ 3A) $(-1,\frac{\pi}{2}), (1,\frac{3\pi}{2})$ 6A) \$26

Team E) 188 : 315

Let
$$BF = l$$
, $BC = w$.
 $\Delta DIJ \sim \Delta GIH \Rightarrow \frac{7}{9.8} = \frac{25}{w - (7 + 24)}$
 $\Rightarrow \frac{70}{98} = \frac{5}{7} = \frac{25}{w - 31} \Rightarrow 5w = 175 + 155 \Rightarrow w = 66, GI = 35$
 $\Delta DIJ \sim \Delta BIF \Rightarrow \frac{7}{l} = \frac{24}{w - 24} = \frac{24}{42} = \frac{4}{7} \Rightarrow 4l = 49 \Rightarrow l = \frac{49}{4}$
The areas of the trapezoids are: $|GIDE| = \frac{1}{2} \cdot 7 \cdot (35 + 59) = 7 \cdot 47$,

$$|BIJC| = \frac{1}{2} \cdot \frac{49}{4} \cdot (24 + 66) = \frac{49 \cdot 90}{8}.$$

Thus, the required ratio is $\frac{7 \cdot 47}{\frac{49 \cdot 90}{8}} = \frac{7 \cdot 8 \cdot 47}{49 \cdot 90} = \frac{4 \cdot 47}{7 \cdot 45} \Rightarrow \underline{188:315}.$

