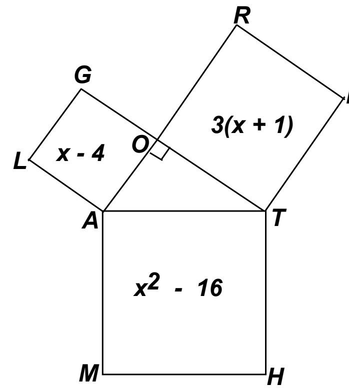
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - October 2021
Practice Problems

1A) A (spherical) marble has a surface area of 216π square units. This marble rolls into the corner of a room, i.e., is tangent to the floor and two walls which are perpendicular to each other and the floor. Compute the distance from the center of the marble to the point where the floor and the two walls intersect.

2A) ΔOAT is a right triangle with hypotenuse $\overline{AT}$. The areas of the squares drawn on the three sides of ΔOAT are given in the diagram at the right. Compute x.



3A) If $\frac{x}{5}-4=2(x+3)$, compute $\lceil x \rceil$, the smallest integer greater than or equal to x.

4A) By how much does $41\frac{2}{3}\%$ of 3600 exceed $62\frac{1}{2}\%$ of 1600?

5A) For how many integer values of x do the values of $2x^2-1984x$ and $\left|2x^2-1984x\right|$ differ in sign?
Note: Expressions that evaluate to zero are considered to have the same sign.

6A) Evaluate $\frac{7}{9}+\frac{1}{16}+\sqrt{\frac{7}{9}+\frac{1}{16}}$ as a reduced ratio of integers.

Team E)

For integer constants A and B, where $B>A>0$, $\begin{cases}|x+8|\geq A\\|12-x|<B\end{cases}$.

The <u>minimum</u> number of integer solutions is k, and the sum of these k solutions is S.
Compute the ordered pair (k, S).

Answers:

1A)	$9\sqrt{2}$	4A)	500
2A)	$2+\sqrt{19}$	5A)	991
3A)	-5	6A)	$\frac{253}{144}$

Team E) (3, 36)

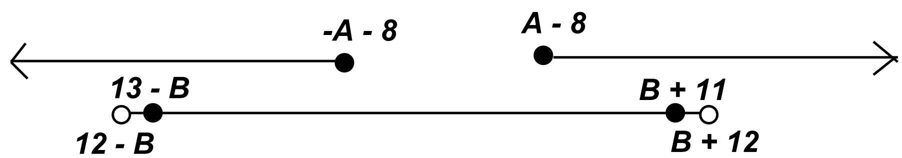
For integer constants A and B, where $B > A > 0$, $\begin{cases} |x+8| \ge A \\ |12-x| < B \end{cases}$.

The <u>minimum</u> number of integer solutions is k, and the sum of these k solutions is S. Compute the ordered pair (k, S).

$|x+8| \ge A \Leftrightarrow x+8 \le -A$ or $x+8 \ge A \Leftrightarrow \boxed{x \le -A-8 \text{ or } x \ge A-8}$ (two rays)

$|12-x| < B \Leftrightarrow -B < 12-x < B \Leftrightarrow B > x-12 > -B \Leftrightarrow \boxed{B+12 > x > 12-B}$ (an open-ended segment)

Graphically, the solution set is



Adding 1 to $12-B$, and subtracting 1 from $B+12$, we have the smallest possible integer solution $13-B$, and the largest possible integer solution $B+11$.

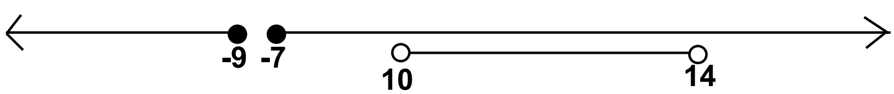
Taking the intersection of the two inequalities, the <u>integer</u> solutions of the system can be found as follows: $13-B \le -A-8 \Leftrightarrow A-B \le -21 \Leftrightarrow \boxed{B-A \ge 21}$ and $A-8 \le B+11 \Leftrightarrow \boxed{B-A \ge -19}$.

The latter condition is always satisfied, and this guarantees that the right-pointing ray always intersects the open-ended segment.

If the former condition is satisfied (and this is not necessarily the case, given the initial conditions), then the ray pointing to the left is guaranteed to intersect the open ended segment.

All the integer solutions lie on the segment. The shorter the segment, the fewer the number of integer solutions. Since the smallest possible value of B is 2 (for A = 1), the open-ended segment is defined by $10 < x < 14$, producing integer solutions 11,12, and 13, for a sum of <u>**36**</u>.

Below is the actual graph of the system with the minimum number of integer solutions:



Challenge: Assuming $B > A > 0$,

Can you determine a general formula for k?

Can you develop an expression for the sum of these integer solutions strictly in terms of the the unknown constants A and B?