MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2019
ROUND 1 VOLUME & SURFACES

ANSWERS

A) ________________________________

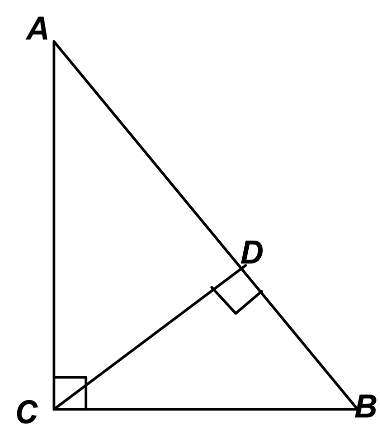
B) ________________________________

C) ________________________________

A) The ratio of the area of a great circle of a sphere to the circumference of a great circle of the sphere is the same as the ratio of the surface area of the sphere to the volume of the sphere. Compute the radius of such a sphere.

B) A pyramid has a square base with area of A^2, and congruent lateral faces, each with area $\frac{A^2}{3}$.
The total surface area of the pyramid is 84.
Compute the height of this pyramid.

C) $\overline{CD} \perp \overline{AB}$.
Point C is rotated about segment $\overline{AB}$, creating two cones.
If $AC = 4$ and $BC = 3$, the positive difference between the volumes of the two cones is $k\pi$. Compute k.

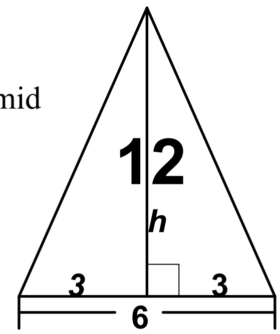


MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2019 SOLUTION KEY

Round 1

A) A great circle of a sphere is any circle with the same center and radius as the sphere.

Thus, the required ratio is $\dfrac{\pi r^2}{2\pi r}=\dfrac{r}{2}=\dfrac{4\pi r^2}{\frac{4}{3}\pi r^3}=\dfrac{3}{r}$. Cross-multiplying, we have $r=\underline{\sqrt{\mathbf{6}}}$.

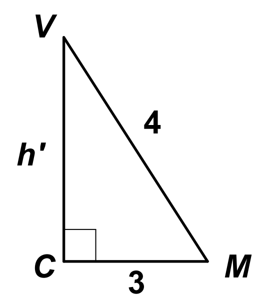


B) $A^2+4\cdot\dfrac{A^2}{3}=84\Rightarrow 7A^2=3\cdot 84\Rightarrow A^2=36\Rightarrow A$, the edge of the base of the pyramid and the base of each isosceles face, is 6. The area of each face is

$\dfrac{A^2}{3}=\dfrac{36}{3}=12=\dfrac{1}{2}bh=\dfrac{1}{2}6h\Rightarrow h=4$.

Consider the right triangle with vertices at the vertex V of the pyramid, the center C of the base, and the midpoint M of the base of any isosceles face. VM is both the altitude of a face and the slant height of the pyramid.

Clearly, the height of the pyramid is $h'=\sqrt{4^2-3^3}=\underline{\sqrt{\mathbf{7}}}$.



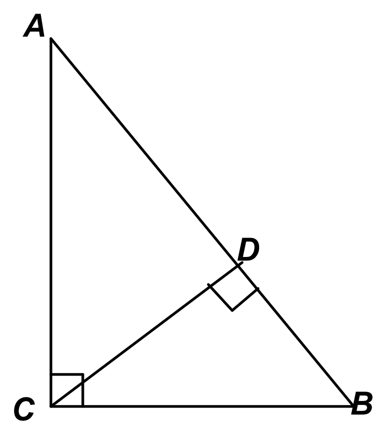
C) $AB=5,\ CD=\dfrac{12}{5}$

By similar triangles ($\Delta ABC\sim\Delta CBD$),

$\dfrac{AB}{CB}=\dfrac{BC}{BD}\Leftrightarrow\dfrac{5}{3}=\dfrac{3}{BD}\Rightarrow BD=\dfrac{9}{5},AD=\dfrac{16}{5}$

Thus, the radius of both cones is $\dfrac{12}{5}$, while the heights are $\dfrac{9}{5}$ and $\dfrac{16}{5}$, respectively.

$V=\dfrac{\pi}{3}\left(\dfrac{12}{5}\right)^2\left(\dfrac{16}{5}\right)-\dfrac{\pi}{3}\left(\dfrac{12}{5}\right)^2\left(\dfrac{9}{5}\right)=\dfrac{\pi}{3}\left(\dfrac{12}{5}\right)^2\cdot\dfrac{7}{5}=\dfrac{336}{125}\pi\Rightarrow k=\underline{\dfrac{\mathbf{336}}{\mathbf{125}}}$.



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2019
ROUND 2 PYTHAGOREAN RELATIONS IN RECTILINEAR FIGURES

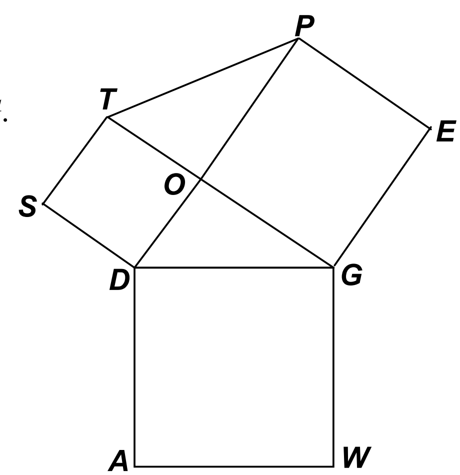
ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________

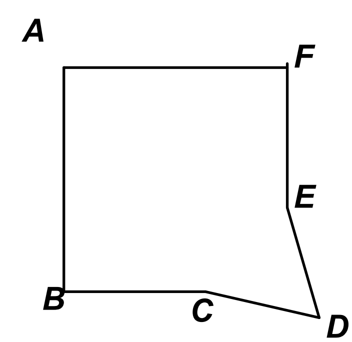
A) Quadrilaterals *STOD*, *POGE* and *DAWG* are squares.
Points *P*, *O*, and *D* are collinear.
If the areas of *STOD* and *POGE* are 20 and 48, respectively, compute the sum of the areas of triangle *TOP* and square *DAWG*.



B) Compute the <u>largest integer</u> that is smaller than the square root of the difference between the integers in cell I2 and cell G3.

	1	**2**	**3**
A	7	24	25
B	9	40	41
C	11	60	61
D	13	84	85
E			

C) Given: $\overline{AB}$, $\overline{EF} \perp \overline{AF}$, $\overline{AB} \perp \overline{BC}$, $AB = AF = 4$, $BC = FE = 3$.
ΔCED is an equilateral triangle.
Compute the area of *ABCDEF*.



MASSACHUSETTS MATHEMATICS LEAGUE
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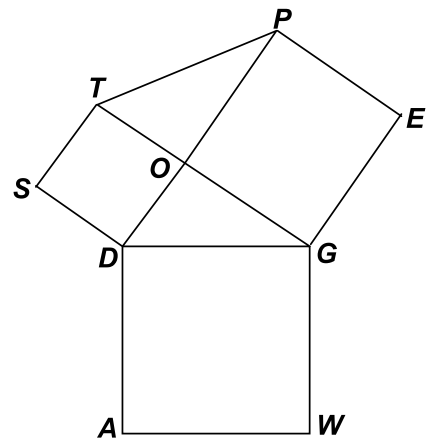
Round 2

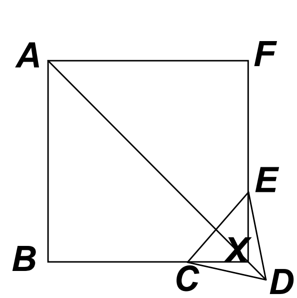


A) Since P, O, and D are collinear, ΔDOG must be a right triangle. By the Pythagorean Theorem, the area of square $DAWG$ must be $20 + 48 = 68$. Since $TO = \sqrt{20} = 2\sqrt{5}$ and $OP = \sqrt{48} = 4\sqrt{3}$, the area of $\Delta TOP = \frac{1}{2} \cdot 2\sqrt{5} \cdot 4\sqrt{3} = 4\sqrt{15}$. Thus, the required sum is $\underline{\mathbf{4\sqrt{15} + 68}}$.

B) The gap between the entries in column 1 is 2, while the gap in column 2 is increasing by 4. In every row, the entry in column 3 is 1 more than the entry in column 2. $\sqrt{264 - 181} = \sqrt{83} > \sqrt{81} = \underline{\mathbf{9}}$.

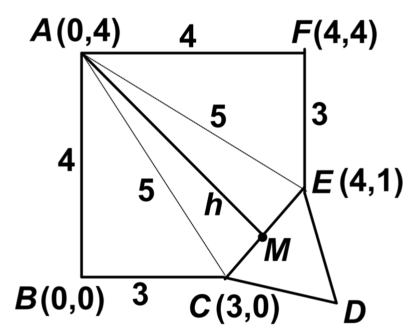
	1	2	3
A	7	24	25
B	9	40	41
C	11	60	61
D	13	84	85
E	15	112	113
F	17	144	145
G	19	180	181
H	21	220	221
I	23	264	265

C) $BC = 3 \Rightarrow CX = 1 \Rightarrow |\Delta CXE| = \frac{1}{2} \Rightarrow |ABCEF| = 16 - \frac{1}{2} = \frac{31}{2}$



$CE = \sqrt{2} \Rightarrow |\Delta CED| = \dfrac{\left(\sqrt{2}\right)^2 \cdot \sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

Thus, the required area is $\underline{\mathbf{\dfrac{31+\sqrt{3}}{2}}}$ or $\underline{\mathbf{15.5 + \dfrac{\sqrt{3}}{2}}}$.



Alternately, let point B be the origin and points A, C, E and F have coordinates as indicated in the diagram at the right.

$CE = \sqrt{2} \Rightarrow CM = \dfrac{\sqrt{2}}{2}$. $h^2 + \left(\dfrac{\sqrt{2}}{2}\right)^2 = 5^2 \Rightarrow h = \sqrt{25 - \dfrac{1}{2}} = \dfrac{7}{\sqrt{2}}$.

Thus, the area of pentagon $ABCEF$ is

$2\left(\dfrac{1}{2} \cdot 3 \cdot 4\right) + \dfrac{1}{2} \cdot \sqrt{2} \cdot \dfrac{7}{\sqrt{2}} = 12 + \dfrac{7}{2} = \dfrac{31}{2}$, and the same result follows.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2019
ROUND 3 ALGEBRA 1: LINEAR EQUATIONS

ANSWERS

A) ________________________________

B) ________________________________

C) ________________________________

A) My term grade was determined by my 5 test scores and my final exam. Each test was based on 100 points, but the final exam counted twice. If my average for the first 5 tests was 88 and my term average was 90, what was my score on the final exam?

B) Given: $\begin{cases} \dfrac{x}{y} = k \\ x - 6y = 2019 \end{cases}$

If x, y, and k are positive integers and $y \neq 1$, compute the value of $x + y$ <u>closest to</u> 2019.

C) Given: $\begin{cases} x + 2y = a \\ x - 4y = b \end{cases}$, where a and b are positive integers, and x and y are integers,

Compute $x + y$, if $a : b = 7 : 11$ and $a + b$ has a <u>minimum</u> value.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2019 SOLUTION KEY

Round 3

A) $\dfrac{5\cdot 88+2F}{7}=90 \Rightarrow F=\dfrac{630-440}{2}=\dfrac{190}{2}=\underline{\mathbf{95}}$.

B) Knowing the prime factorization of 2019 is helpful.

Substituting ky for x in the second equation, and solving for y, we have $y=\dfrac{2019}{k-6}=\dfrac{3\cdot 673}{k-6}$.

$k-6=1,\ 3,\ 673,\ 2019 \Rightarrow (k,y)=(7,2019),\ (9,673),\ (679,3),\ \cancel{(2025,1)}$.

Substituting for x, $x+y=ky+y=y(k+1)=$ 16152, 6730, $\underline{\mathbf{2040}}$.

C) If $a=7c$, $b=11c$, then, subtracting, we have $\begin{cases} x+2y=7c \\ x-4y=11c \end{cases} \Rightarrow 6y=-4c$ or $y=-\dfrac{2}{3}c$.

To minimize $a+b$, we must minimize c. To produce an integer value for y, we take $c=3$
$\Rightarrow \begin{cases} a=21 \\ b=33 \end{cases}$.

$y=-2 \Rightarrow x-4=21 \Rightarrow x=25$.

Thus, $x+y$ is $\underline{\mathbf{23}}$.

MASSACHUSETTS MATHEMATICS LEAGUE
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ROUND 4 ALGEBRA 1: FRACTIONS & MIXED NUMBERS

ANSWERS

A) ________________________________

B) ________________________________

C) ________________________________

A) A sequence of fractions is generated by these rules:

- If the numerator is even, increase it by 3, otherwise decrease it by 1.
- If the denominator is even, increase it by 1, otherwise, decrease it by 3.

Start with $\frac{8}{13}$. Eventually an integer k is produced. Compute k.
While searching for this integer, do <u>not</u> reduce the fractions.

B) $\frac{6}{5}$ of A equals $83\frac{1}{3}\%$ of B, for positive integers A and B.
$\frac{A}{B}$ and $\frac{B}{A}$ are reduced fractions. Exactly one of these fractions is equal to a terminating decimal. Compute this terminating decimal value.

C) Given: $\frac{Kx+J}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}$, where J and K are <u>consecutive</u> <u>odd</u> integers .
If $J < K < 33$, compute the <u>largest</u> value of $A+B$ that is prime.

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Round 4

A) $\frac{8}{13}, \frac{11}{10}, \frac{10}{11}, \frac{13}{8}, \frac{12}{9}, \frac{15}{6}, \frac{14}{7} = \underline{\mathbf{2}}$.

B) $83\frac{1}{3}\% = \frac{5}{6} \Rightarrow \frac{6}{5}A = \frac{5}{6}B \Rightarrow \frac{A}{B} = \frac{25}{36}$.

Terminating decimals have denominators containing only factors which are powers of 2 or powers of 5. Thus, $\frac{25}{36}$ equals a repeating decimal, and $\frac{36}{25}$ equals a terminating decimal.
$\frac{1}{25} = 0.04 \Rightarrow \frac{36}{25} = \underline{\mathbf{1.44}}$.

C) $\frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{A(x-1)+B}{(x-1)^2} = \frac{Ax+(B-A)}{(x-1)^2} = \frac{Kx+J}{(x-1)^2}$.
Therefore, $A = K$ and $B - A = J \Rightarrow B = A + J = K + J$.
Since $A + B = 2K + J$, and K and J are consecutive odd integers, and $J < K < 33$, we start with $(K, J) = (31, 29) \Rightarrow 62 + 29 = 91 = 7 \cdot 13$, rejected.
$(29, 27) \Rightarrow 85$, rejected. $(27, 25) \Rightarrow \underline{\mathbf{79}}$ Bingo!

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2019
ROUND 5 INEQUALITIES & ABSOLUTE VALUE

ANSWERS

A) ______________________________

B) (____________ , ____________)

C) ______________________________

A) For integers x, y, and z, $7x-3>20$, $5y-8>19$, and $x+y+z=12$.
Compute the <u>maximum</u> value of z.

B) The minimum and maximum values of x for which $y=f\left(x\right)=\sqrt{40+39x-40x^2}$ defines a real-valued function of a real variable are m and M, respectively.
Compute the ordered pair $\left(m,M\right)$.

C) Solve for x. $\dfrac{4}{x+2}-\dfrac{3}{x-1}\geq 0$

MASSACHUSETTS MATHEMATICS LEAGUE
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Round 5

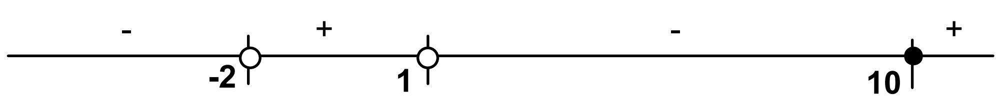
A) $7x-3>20 \Rightarrow 7x>23 \Rightarrow x\geq 4$.
$5y-8>19 \Rightarrow 5y>27 \Rightarrow y\geq 6$.
$x+y\geq 10$ and $x+y+z=12 \Rightarrow z_{\text{max}}=\underline{\mathbf{2}}$.

B) The radicand $40+39x-40x^2=(8-5x)(5+8x)$ must be nonnegative, implying $-\frac{5}{8}\leq x\leq \frac{8}{5}$.

Thus, $(m,M)=\underline{\left(-\mathbf{\frac{5}{8}},\mathbf{\frac{8}{5}}\right)}$.

C) $\dfrac{4}{x+2}-\dfrac{3}{x-1}\geq 0 \Leftrightarrow \dfrac{4(x-1)-3(x+2)}{(x+2)(x-1)}\geq 0 \Leftrightarrow \dfrac{x-10}{(x+2)(x-1)}\geq 0$

Critical values at -2, 1 and 10.
As $x\to -\infty$ (to the left), the values of all three binomials are negative.
As $x\to +\infty$ (to the right), the values of all three binomials are positive.
Moving from left to right, as each critical value is passed, the value of exactly one more binomial becomes positive.



Thus, the inequality is satisfied for $\underline{\mathbf{-2<x<1 \text{ or } x\geq 10}}$.

MASSACHUSETTS MATHEMATICS LEAGUE
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ROUND 6 ALGEBRA 1: EVALUATIONS

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________________

A) Given: $a \copyright b = \dfrac{2ab}{a+b}$

If $x \copyright (3x) = 6$, compute $x \copyright 6$, provided $x \neq 0$.

B) I estimated 45% of 75 to be equal to 50% of 70 (which is much easier to calculate). My estimate is larger than the correct answer by *k*%. Compute *k to the nearest integer*.

A reminder: If the correct number of jelly beans in a jar is 800 and I estimated the number to be 1000, I have overestimated by 25%.

C) Determine the <u>smallest</u> positive 3-digit number *N* with distinct digits, such that *N* is equal to 6 less than 5 times the square of the sum of its digits.

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Round 6

A) $x \copyright (3x) = 6 \Leftrightarrow \frac{2x(3x)}{x+3x} = \frac{6x^2}{4x} = 6 \Rightarrow \frac{x}{4} = 1 \Rightarrow x = 4$

$4 \copyright 6 = \frac{2 \cdot 4 \cdot 6}{4+6} = \frac{48}{10} = \underline{\mathbf{4.8}}$.

B) 45% of $75 = \frac{45}{100} \cdot 75 = \frac{45 \cdot 3}{4} = \frac{135}{4} = 33.75$.

My estimate is 35 - an overestimate by 1.25.

The percent error is $\frac{1.25}{33.75} = \frac{125}{3375} = \frac{5}{135} = \frac{1}{27}$.

Converting to a percent (over 100), we have $\frac{1}{27} = \frac{k}{100} \Rightarrow k = \frac{100}{27} = 3\frac{19}{27} \Rightarrow k = \underline{\mathbf{4}}$.

C) We require that $100h + 10t + u = 5(h+t+u)^2 - 6$.

If we let $n = (h+t+u)^2$, then $5n - 6$ must be a 3-digit integer.

Since $5(4^2) - 6 = 74$ and $5(5^2) - 6 = 119$, the sum of the digits must be at least 5.

If digitsum of N is	$5n$-6 must be	Verdict
5	119	rejected (digitsum = 11)
6	174	rejected (digitsum = 12)
7	239	rejected (digitsum = 14)
8	314	Bingo! (digitsum = 8)

The smallest three-digit number satisfying the stated conditions is **314**.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2019
ROUND 7 TEAM QUESTIONS

ANSWERS

A) ____________________________ D) ____________________________

B) ____________________________ E) (_____ , _____ , _____ , _____)

C) ____________________________ F) ____________________________

A) Let A denote the area of the base of a right cone with a circular base. Let L denote its lateral surface area. If $\frac{A}{L}=0.28$ and the total surface area of the cone is 896π, compute the area of a cross-section of this cone 32 units from the base.

B) How many distinct Pythagorean triples (a, b, c) are there for which $0<a<b<c$ and $a=2^1\cdot 3^{11}\cdot 5^{35}$?

C) Consider the conversion equation $C=\frac{5}{9}(F-32)$. There are many instances where an integer input value for F produces an integer output value for C. (For example, a comfortable room temperature of $68°F$ corresponds to $20°C$.) Let (F_1, C_1) and (F_2, C_2) be solutions to this equation. Determine the *minimum* positive value of F_1 for which F_1, C_1, F_2, and C_2 are all integers and $F_2=C_1$.

D) The decimal fraction 0.975 can be expressed as the sum of distinct unit fractions. Compute a possible sum of the denominators of these unit fractions.

E) There is a minimum positive integer value of the constant A for which the equation $|5x-A|=x^2-2x$ has <u>exactly three</u> integer solutions s_1, s_2, and s_3, where $s_1<s_2<s_3$. Specify the ordered quadruple (A, s_1, s_2, s_3).

F) For each of the 13 k-values, (k = 3 to 15), the largest possible regular k-sided polygons with sides of integral length are made, each from a spool of wire which is exactly 100 feet long. All the pieces of excess wire are soldered together into one long length, which can then be used to form another regular n-sided polygon with integer side-lengths of at least 2 units. Assume no length is gained or lost in the soldering process. For n = 3 to 15, compute the maximum value of $n+l+r$, where l is the length of one side of the regular n-gon, and r is the length of the leftover piece of wire.

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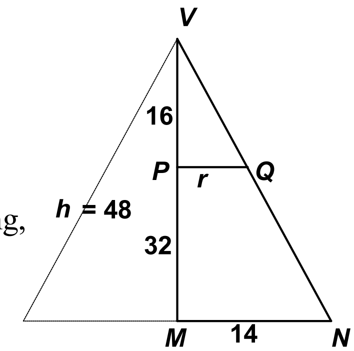
Team Round



A) If l denotes the slant height of the cone, then

$$\frac{A}{L}=\frac{\pi r^2}{\pi rl}=\frac{r}{l}=0.28=\frac{7}{25}\Rightarrow 7l=25r.$$

$\pi r^2+\pi rl=\pi r(r+l)=896\pi$. Multiplying through by 7 and substituting,

we have $\pi r(7r+7l)=7\cdot 896\pi \Rightarrow 32r^2=7\cdot 896=7\cdot 32\cdot 28\Rightarrow$

$r^2=7(28)\Rightarrow r=14$, $l=50,\ h=48$.

Now look at a cross section of the cone through the vertex perpendicular to the base. Let P and M denote the centers of the cross section and base, respectively, and A' the required area. Using similar triangles, $\frac{16}{48}=\frac{r}{14}\Rightarrow r=\frac{14}{3}\Rightarrow A'=\underline{\boldsymbol{\frac{196\pi}{9}}}$.

B) $\left(2^1\cdot 3^{11}\cdot 5^{35}\right)^2+b^2=c^2\Rightarrow 2^2\cdot 3^{22}\cdot 5^{70}=c^2-b^2=(c+b)(c-b)$

The number on the left side of the equation has $(2+1)(22+1)(70+1)$ factors

That's $3\cdot 23\cdot 71=69\cdot 71=(70+1)(70-1)=4900-1=4899$ factors.

That's 2449 pairs of <u>unequal</u> factors P and Q (we assume $P > Q$) and 1 pair of <u>equal</u> factors, i.e., $N=2^1\cdot 3^{11}\cdot 5^{35}$.

Equal factors: $\begin{cases} c+b=N \\ c-b=N \end{cases}\Rightarrow c=N,\ b=0$. This case is rejected.

Unequal factors: $\begin{cases} c+b=P \\ c-b=Q \end{cases}\Rightarrow c=\frac{P+Q}{2},\ b=\frac{P-Q}{2}$

Clearly, P and Q must be both even or both odd (otherwise, b and c would not be integers).
Since the given a-value contains a factor of 2, both P and Q must be even.
Some of the 2449 factor pairs are even-odd pairs and must be discounted.
Selecting from only powers of 3 and 5, we have $(11+1)(35+1)/2=216$ pairs of unequal odd factors.
Multiplying either factor in any pair by 2 gives us two different unwanted even-odd pairs.
Thus, the number of distinct PTs is 2449 –2(216) = <u>**2017**</u>.

C) Start with $(F,C)=(68,20)$. This equation corresponds to a straight line with a slope of $\frac{5}{9}$, where C plays the role of x, and F plays the role of y. As F increases by 9, C increases by 5. Notice that $(68,20)$ fails, since $F=20$ fails to produce an integer value for C.

Examine new pairs until $C-32$ is divisible by 9. (Remember – the C-value is being substituted for F.) These following ordered pairs (77, 25), (86, 30), (95, 35), (104, 40), (113, 45) fail, but $C-32$ is producing a string of different remainders, so we are not stuck in a futile cycle. Bingo (122, 50) is just what the doctor ordered. $F_1=\underline{\mathbf{122}}$. $(122\rightarrow 50\rightarrow 10)$

Team Round - continued

D) $0.975 = \frac{975}{1000} = \frac{1000-25}{1000} = 1 - \frac{1}{40} = \frac{39}{40}$.

Since this is greater than the largest unit fraction $\frac{1}{2}$, we have $\frac{39}{40} = \frac{1}{2} + \left(\frac{39}{40} - \frac{1}{2}\right) = \frac{1}{2} + \frac{19}{40}$.

$\frac{19}{40} > \frac{1}{3}$, since $3 \cdot 19 > 40 \cdot 1$. Thus, $\frac{39}{40} = \frac{1}{2} + \frac{1}{3} + \left(\frac{19}{40} - \frac{1}{3}\right) = \frac{1}{2} + \frac{1}{3} + \frac{17}{120}$.

Clearly, $\frac{17}{120}$ is less than each of $\frac{1}{4}, \frac{1}{5}$, and $\frac{1}{6}$. Is $\frac{1}{7} < \frac{17}{120}$? No , since $17 \cdot 7 = 119 \not> 120$.

Therefore, the next unit fraction must be $\frac{1}{8}$. The sum of the denominators is **73**, since

$\frac{39}{40} = \frac{1}{2} + \frac{1}{3} + \frac{1}{8} + \left(\frac{17}{120} - \frac{1}{8}\right) = \frac{1}{2} + \frac{1}{3} + \frac{1}{8} + \frac{2}{120} = \frac{1}{2} + \frac{1}{3} + \frac{1}{8} + \frac{1}{60}$.

Note : $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{10} = \frac{1}{2} + \frac{1}{5} + \frac{1}{8} + \frac{1}{10} + \frac{1}{20} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{40} = \frac{39}{40}$,

So some alternative answers are **24**, **45**, **51**.

Are there others? Can you describe a method for finding **all** possible answers?

Please send your work to olson.re@gmail.com.

E) Case 1: $5x - A = x^2 - 2x \Leftrightarrow x^2 - 7x + A = 0$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 4A}}{2} \Rightarrow A = 6\ (x = 1, 6),\ 10\ (x = 2, 5)$$

Case 2: $5x - A = -\left(x^2 - 2x\right) \Leftrightarrow x^2 + 3x - A = 0$

$$\Rightarrow x = \frac{-3 \pm \sqrt{9 + 4A}}{2} \Rightarrow A = 4\ (x = 1, -4),\ \ 10\ (x = 2, -5),\ 18\ (x = 3, -6),\ \ldots$$

There are an infinite number of potential A-values in case 2, but we are looking for an A-value in common with case 1 with overlapping x-values

All other A-values except 10 produce exactly two solutions.

Checking $x = 2, \pm 5$ in $|5x - 10| = x^2 - 2x$, we have $\begin{cases} |-25 - 10| = (-5)^2 - 2(-5) = 35 \ \surd \\ |5 \cdot 2 - 10| = 2^2 - 2 \cdot 2 = 0 \ \surd \\ |25 - 10| = 5^2 - 2 \cdot 5 = 15 \ \surd \end{cases}$

Thus, $\left(A, s_1, s_2, s_3\right) = \mathbf{(10, -5, 2, 5)}$.

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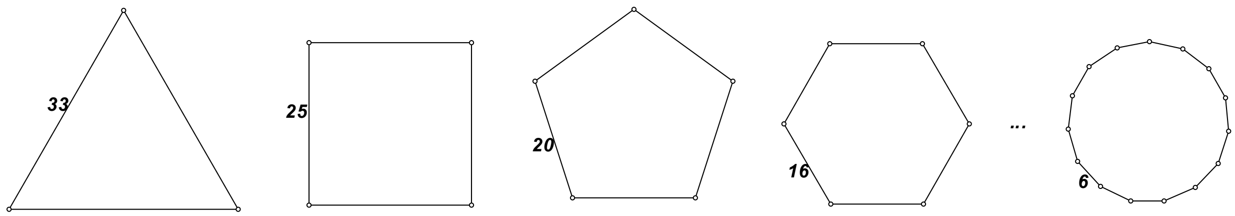
Team Round - continued

F)



k	3	4	5	6	7	8	9	10	11	12	13	14	15
Per	99	100	100	96	98	96	99	100	99	96	91	98	90
Excess	1	0	0	4	2	4	1	0	1	4	9	2	10

The length of the leftover wire is $\boxed{38}$. Surprisingly, the chart at the right shows that a 19-sided polygon with sides of length 2 (leaving a remainder of 0) gives us $n + l + r = 21$ which is *not* the maximum possible value. The maximum sum is associated with the largest remainder.

n	l	r	sum	n	l	r	sum	n	l	r	sum
3	12	2	17	9	4	2	15	15	2	8	25
4	9	2	15	10	3	8	21	16	2	6	24
5	7	3	15	11	3	5	19	17	2	4	23
6	6	2	14	12	3	2	17	18	2	2	22
7	5	3	15	13	2	12	**27**	19	2	0	21
8	4	6	18	14	2	10	26	20	-	-	-

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2019 ANSWERS

Round 1 Geometry - Volume and Surface Area

A) $\sqrt{6}$ B) $\sqrt{7}$ C) $\dfrac{336}{125}$

Round 2 Pythagorean Relations

A) $4\sqrt{15}+68$ B) 9 C) $\dfrac{31+\sqrt{3}}{2}$

Round 3 Linear Equations

A) 95 B) 2040 C) 23

Round 4 Fractions & Mixed numbers

A) 2 B) 1.44 C) 79

Round 5 Absolute value & Inequalities

A) 2 B) $\left(-\dfrac{5}{8}, \dfrac{8}{5}\right)$ C) $-2 < x < 1$ or $x \geq 10$

Round 6 Evaluations

A) 4.8 or $\dfrac{24}{5}$ B) 4 C) 314

Team Round

A) $\dfrac{196\pi}{9}$

B) 2017

C) 122

D) 73

Alternate answers: 24, 45, 51
Check for others!

E) $(10, -5, 2, 5)$

F) 27