

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2017
ROUND 1 COMPLEX NUMBERS (No Trig)

ANSWERS

A) _____

B) (_____ , _____)

C) (_____ , _____)

A) Let $P = \sqrt{-125} \cdot \sqrt{-45}$ and $Q^2 = P$.
Compute Q .

B) Given: $z = 1 + i$

Compute the ordered pair (A, B) , if $A + Bi = \frac{1}{z} + (-z) + \bar{z} + z^2$.

C) Compute the ordered pair of positive integers (m, k) for which $\frac{1+i\sqrt{k}}{m} + \frac{m}{1+i\sqrt{k}} = 1$.

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2017 SOLUTION KEY**

Round 1

A) $P = \sqrt{-125} \cdot \sqrt{-45} = \sqrt{125(-1)} \cdot \sqrt{45(-1)} = (5\sqrt{5} \cdot i)(3\sqrt{5} \cdot i) = 15 \cdot 5 \cdot i^2 = -75$
 $Q^2 = P = -75 \Rightarrow Q = \underline{\pm 5i\sqrt{3}}$.

B) $\frac{1}{z} + (-z) + \bar{z} + z^2 = \frac{1}{1+i} + (-1-i) + (1-i) + (1+i)^2 = \frac{1-i}{2} - 2i + (1+2i-1) = \frac{1-i}{2} \Rightarrow \underline{\left(\frac{1}{2}, -\frac{1}{2}\right)}$.

C) $\frac{1+i\sqrt{k}}{m} + \frac{m}{1+i\sqrt{k}} = \frac{(1+i\sqrt{k})^2 + m^2}{m(1+i\sqrt{k})} = 1 \Rightarrow 1 - k + m^2 + 2i\sqrt{k} = m + mi\sqrt{k}$

Equating the imaginary components, $m = 2$.

Equating the real components and substituting for m , we have $1 - k + 4 = 2 \Rightarrow k = 3$.

Thus, $(m, k) = \underline{(2, 3)}$.

Alternately, let $z = \frac{1+i\sqrt{k}}{m}$. Then: $z + \frac{1}{z} = 1 \Rightarrow z^2 - z + 1 = 0 \Rightarrow z = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$.

$\frac{1+i\sqrt{k}}{m} = \frac{1+i\sqrt{3}}{2} \Rightarrow (m, k) = \underline{(2, 3)}$.

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017
ROUND 2 ALGEBRA 1: ANYTHING

ANSWERS

A) _____

B) _____

C) (_____ , _____)

A) Let $y = \frac{5}{9}(x - 32)$. If both x and y are positive integers, compute the minimum sum $x + y$.

B) Given:
$$\begin{cases} A^3 - B^3 = 296 \\ \frac{1}{B} + \frac{1}{30} = \frac{1}{5} \end{cases}$$
 for integers A and B .

Compute $AB^2 - A^2B$.

C) At 55 mph, I will arrive at my destination in 18 minutes.
If I increase my speed by k mph, I will arrive at my destination in 15 minutes.
Compute the ordered pair (k, d) , where d denotes the distance to my destination (in miles).

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2017 SOLUTION KEY**

Round 2

A) Making $x - 32$ a positive multiple of 9 and as small as possible, we have $41 - 32 = 9$ which produces $y = 5$ and $x + y = \underline{46}$.

$$\text{B) } 30B \cdot \left(\frac{1}{B} + \frac{1}{30} = \frac{1}{5} \right) \Rightarrow 30 + B = 6B \Rightarrow B = 6$$

$$A^3 - B^3 = 296 \Rightarrow A^3 = 296 + 216 = 512 = 2^9 \Rightarrow A = 2^3 = 8$$

$$AB^2 - A^2B = AB(B - A) = 8 \cdot 6 \cdot -2 = \underline{-96}.$$

$$\text{C) } R \cdot T = D \Rightarrow d = 55 \cdot \frac{18}{60} = 55 \cdot \frac{3}{10} = \frac{33}{2} = 16.5$$

$$R_{\text{new}} \cdot \frac{15}{60} = 16.5 \Rightarrow R_{\text{new}} = 66 \Rightarrow k = 11$$

Thus, $(k, d) = \underline{(11, 16.5)}$.

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017
ROUND 3 PLANE GEOMETRY: AREAS OF RECTILINEAR FIGURES**

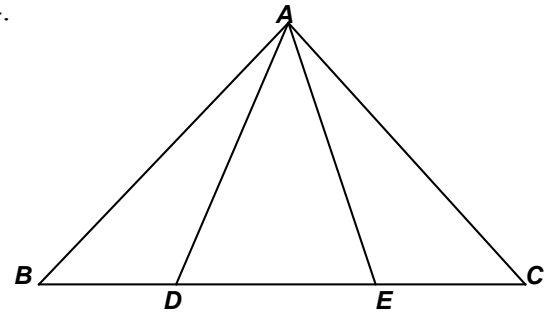
ANSWERS

A) _____

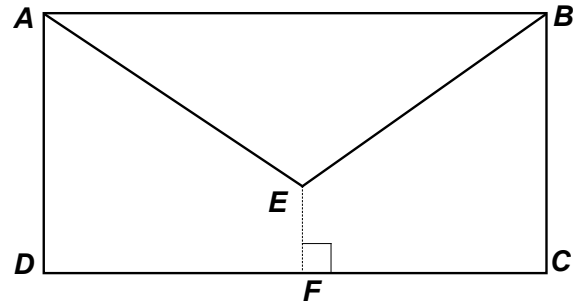
B) _____

C) _____ : _____

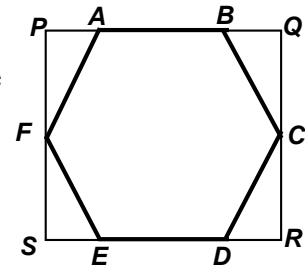
- A) In isosceles triangle ABC , $AB = AC = 12$ and $BD = CE = 4$.
If $\text{area}(\triangle BAD) : \text{area}(\triangle ADE) = 2 : 3$, compute BC .



- B) The diagram at the right shows the back of a 4" x 9" envelope. If $AE = BE = 5.1$, compute the area of trapezoid $BCFE$.



- C) Consider regular hexagon $ABCDEF$ and rectangle $PQRS$, as indicated in the diagram at the right, where $AB = 2 \cdot AP$. Compute the ratio of the area of the hexagon to the area of the rectangle.



**MASSACHUSETTS MATHEMATICS LEAGUE
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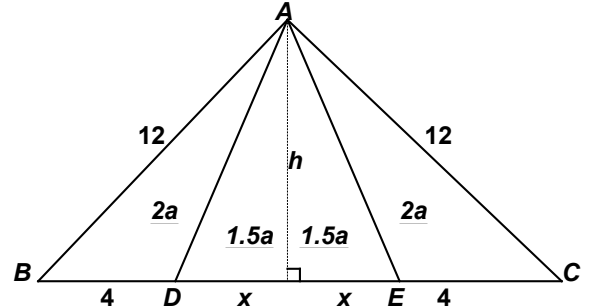
Round 3

A) $\text{area}(\triangle ACE) = \frac{1}{2} \cdot h \cdot 4 = 2h.$

$$\text{area}(\triangle ADE) = 2 \left(\frac{1}{2} \cdot h \cdot x \right) = hx.$$

$$\frac{2h}{hx} = \frac{2}{3} \Rightarrow x = 3 \Rightarrow BC = 4 + 4 + 6 = \underline{14}.$$

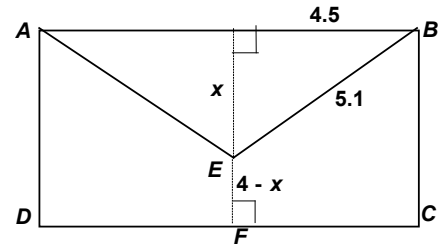
[The information that $AB = AC = 12$ was irrelevant.]



B) Thinking Pythagorean Triple,

$$(_, 4.5, 5.1) = \frac{3}{10}(_, 15, 17) \Rightarrow x = \frac{3}{10} \cdot 8 = 2.4 \Rightarrow EF = 1.6.$$

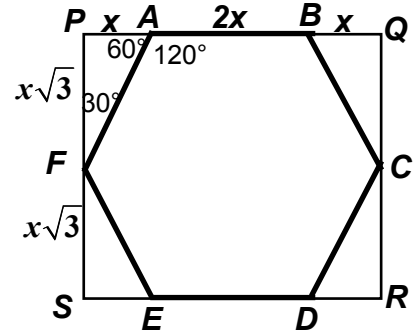
$$\text{Thus, } |BCFE| = \frac{1}{2} \cdot 4.5(4 + 1.6) = 4.5(2.8) = \underline{12.6}.$$



C) The area of the rectangle is $(4x)(2x\sqrt{3}) = 8x^2\sqrt{3}.$

$$\text{The area of the hexagon is } 8x^2\sqrt{3} - 4 \left(\frac{1}{2} x \cdot x\sqrt{3} \right) = 6x^2\sqrt{3}.$$

Thus, the required ratio is $6 : 8 = \underline{3 : 4}.$



MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017
ROUND 4 ALG 1: FACTORING AND ITS APPLICATIONS

ANSWERS

A) _____

B) _____

C) _____

A) Given: $5y(2y - 3x) = 3x(3x - 2y)$

y may be expressed in terms of x , namely $y = kx$, for a rational constant k .

Compute all possible values of k .

B) Compute the sum of the distinct primes in the prime factorization of $6^6 - 2^3 \cdot 3^3$

C) Solve for x . $x^4 = 25x^2 - 144$

Hint: Write a trinomial as the difference of perfect squares.

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017 SOLUTION KEY**

Round 4

A) Noticing that the binomials are opposites of each other makes the job simpler.

$$5y(2y - 3x) = 3x(3x - 2y) \Leftrightarrow 5y(2y - 3x) = -3x(2y - 3x) \Leftrightarrow 5y(2y - 3x) + 3x(2y - 3x) = 0$$

Factoring out the common binomial factor,

$$(2y - 3x)(5y + 3x) = 0 \Rightarrow y = \frac{3}{2}x, -\frac{3}{5}x \Rightarrow k = \frac{3}{2}, -\frac{3}{5} \text{ (or } \underline{1.5}, \underline{-0.6})$$

Any order is allowed and the lead zero is not required.

B) $6^6 - 2^3 \cdot 3^3 = (6^3)^2 - 6^3 = 6^3(6^3 - 1) = 6^3(215) = 2^3 \cdot 3^3 \cdot 5 \cdot 43 \Rightarrow 2 + 3 + 5 + 43 = \underline{53}$

C) $x^4 = 25x^2 - 144 \Leftrightarrow x^4 - 25x^2 - 144 = (x^4 - 24x^2 + 144) - x^2 = (x^2 - 12)^2 - x^2$
 $= (x^2 - 12 + x)(x^2 - 12 - x) = (x^2 + x - 12)(x^2 - x - 12) = (x + 4)(x - 3)(x + 3)(x - 4 = 0)$
 $\Rightarrow x = \pm 3, \pm 4$

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017
ROUND 5 TRIG: FUNCTIONS OF SPECIAL ANGLES**

ANSWERS

A) _____

B) _____

C) _____

A) Given: $\left(\cot 135^\circ + \csc\left(\frac{-5\pi}{6}\right) + \cot\left(\frac{11\pi}{6}\right) \right)^3 = a + b\sqrt{3}$, where a and b are rational numbers.

Compute $b - a$.

B) Compute all values of x over the open interval $0 < x < \pi$, for which the expression

$\frac{1}{\cos^2 x - 3\sin^2 x}$ is undefined.

C) Compute all values of x (in radians) for which $2\sin\left(4x + \frac{5\pi}{12}\right) - 1 = 0$ and $0 \leq x < \pi$

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017 SOLUTION KEY**

Round 5

$$\begin{aligned} \text{A) } & \left(\cot 135^\circ + \csc\left(\frac{-5\pi}{6}\right) + \cot\left(\frac{11\pi}{6}\right) \right)^3 \\ & = (-1 - 2 - \sqrt{3})^3 = -(\sqrt{3} + 3)^3 = -\left[(\sqrt{3})^3 + (3)(\sqrt{3})^2(3) + 3(\sqrt{3})(9) + 27 \right] \\ & = -\left[3\sqrt{3} + 27 + 27\sqrt{3} + 27 \right] = -54 - 30\sqrt{3} \Rightarrow b - a = \underline{24}. \end{aligned}$$

$$\begin{aligned} \text{or } & -(\sqrt{3} + 3)^3 = -(\sqrt{3} + 3)(\sqrt{3} + 3)^2 = -(\sqrt{3} + 3)(6\sqrt{3} + 12) = -(18 + 36 + 30\sqrt{3}) \\ & = -54 - 30\sqrt{3} \text{ etc.} \end{aligned}$$

B) The expression $\frac{1}{\cos^2 x - 3\sin^2 x}$ will be undefined if and only if $\cos^2 x - 3\sin^2 x = 0$.

$$\cos^2 x - 3\sin^2 x = 0 \Leftrightarrow \cos^2 x = 3\sin^2 x = 3(1 - \cos^2 x) \Leftrightarrow 4\cos^2 x = 3$$

$$\Rightarrow \cos x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \underline{\underline{\frac{\pi}{6}, \frac{5\pi}{6}}} \text{ (over the stated interval).}$$

$$\text{C) } 2\sin\left(4x + \frac{5\pi}{12}\right) - 1 = 0 \Rightarrow \sin\left(4x + \frac{5\pi}{12}\right) = \frac{1}{2} \Rightarrow 4x + \frac{5\pi}{12} = \begin{cases} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{cases} + 2n\pi \Rightarrow x = \frac{1}{4} \cdot \begin{cases} \frac{\pi}{6} - \frac{5\pi}{12} \\ \frac{5\pi}{6} - \frac{5\pi}{12} \end{cases} + \frac{n\pi}{2}$$

$$x = \frac{1}{4} \cdot \begin{cases} -\frac{\pi}{4} \\ \frac{5\pi}{12} \end{cases} + \frac{n\pi}{2} = \begin{cases} -\frac{\pi}{16} \\ \frac{5\pi}{48} \end{cases} + \frac{n\pi}{2} = \begin{cases} \frac{8n-1}{16}\pi \\ \frac{24n+5}{48}\pi \end{cases} \quad \text{Letting } n = 0, 1, 2, \text{ we have}$$

$$\Rightarrow \underline{\underline{\frac{5\pi}{48}, \frac{7\pi}{16}, \frac{29\pi}{48}, \frac{15\pi}{16}}} \text{ This listing is in order of increasing magnitude.}$$

Order does NOT matter.

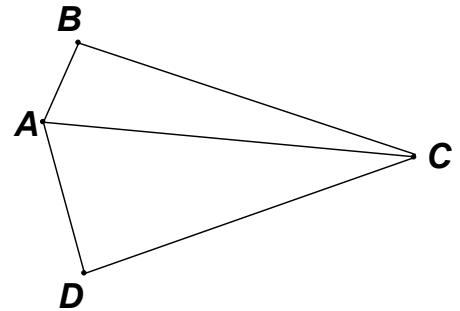
**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017
ROUND 6 PLANE GEOMETRY: ANGLES, TRIANGLES AND PARALLELS**

ANSWERS

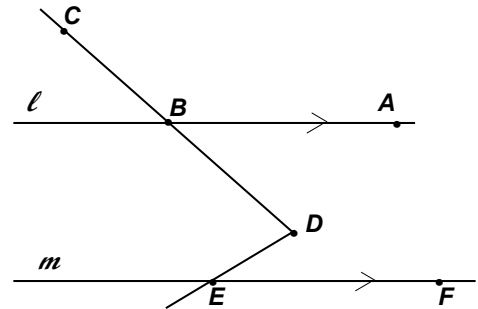
- A) _____
 B) (_____ , _____)
 C) _____

A) A base angle in an isosceles triangle with an obtuse vertex angle measures $(100 - 3x)^\circ$.
 Compute the minimum integer value of x .

B) A convex quadrilateral $ABCD$ has interior angles in the ratio of $1 : 2 : 3 : 4$.
 Suppose A is the largest angle and the smallest angle is the opposite angle.
 Diagonal \overline{AC} divides $\angle BAD$ into two smaller angles whose measures are in a $5 : 4$ ratio.
 \overline{AC} divides $\angle BCD$ into two smaller angles whose measures are in a simplified ratio of $m : n$, where $m < n$. Compute the ordered pair (m, n) .



C) Given: $l \parallel m$, $m\angle ABD = x^2 - 7$
 $m\angle BDE = 11x + 1$
 $m\angle DEF = 5x + 1$
 Compute $m\angle ABC$.



**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017 SOLUTION KEY**

Round 6

A) $A = 180 - 2(100 - 3x) = 6x - 20 > 90 \Rightarrow x > \frac{55}{3} \Rightarrow x_{\min} = \left\lceil 18\frac{1}{3} \right\rceil = \underline{19}$.

B) $10x = 360 \Leftrightarrow x = 36 \Rightarrow$ the largest angle measures 144° and the smallest 36° .

Either $(m\angle B, m\angle D) = (2x, 3x) = (72, 108)$

or, possibly, vice versa.

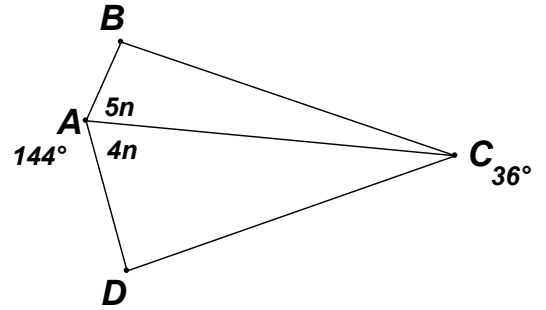
Suppose $m\angle BAC = 5n$ and $m\angle CAD = 4n$. Then:

$9n = 144 \Rightarrow n = 16 \Rightarrow (m\angle BAC, m\angle CAD) = (80, 64)$

$\Rightarrow (m\angle B, m\angle D) = (72, 108)$

$\Rightarrow (m\angle BCA, m\angle DCA) = (28, 8)$

Thus, $(m, n) = \underline{(2, 7)}$.



C) Draw line n parallel to l and m .

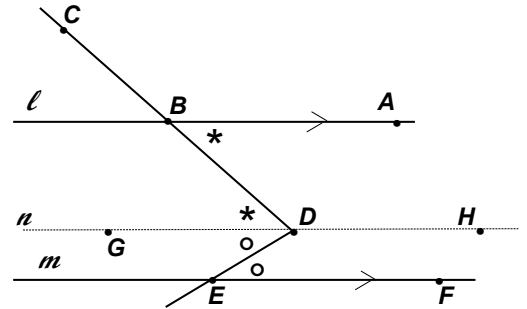
$m\angle EDG = m\angle DEF = 5x + 1$

$m\angle BDG = m\angle ABD = x^2 - 7$

$m\angle BDE = 11x + 1 = * + \circ = (x^2 - 7) + (5x + 1)$

$\Leftrightarrow x^2 - 6x - 7 = (x + 1)(x - 7) = 0 \Rightarrow x = \cancel{1}, 7$

$x = 7 \Rightarrow m\angle ABD = 7^2 - 7 = 42 \Rightarrow m\angle ABC = 180 - 42 = \underline{138^\circ}$.



**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017
ROUND 7 TEAM QUESTIONS**

ANSWERS

- A) _____ D) _____
 B) _____ E) _____
 C) _____ F) _____

A) If $\sqrt{-17-144i} = x + yi$, where x and y are real numbers, compute all possible values of $\frac{x^2}{y}$.

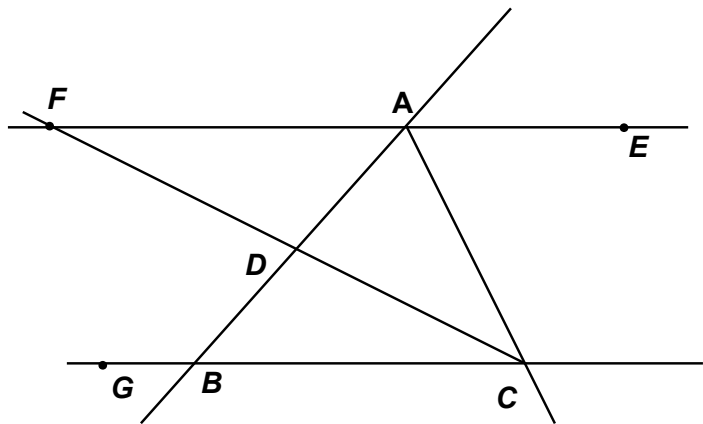
B) The product of two numbers is 2. The sum of the same numbers is 1. Compute the sum of the cubes of these numbers.

C) In $\triangle ABC$, $AC = 4$ and $BC = 6$. If the area of $\triangle ABC$ is $3\sqrt{15}$, compute both possible lengths of \overline{AB} .

D) How many factors of 4158000 are also factors of 122850?

E) Regular octagon $A_1A_2\dots A_8$ is inscribed in an origin-centered circle. Its area is $50\sqrt{2}$. For at least one integer value of i , where $0 < i < 8$, $\overline{A_iA_{i+1}}$ is parallel to the x -axis. How many possible ordered pairs (x, y) are there for vertex A_8 , where x and y are not both positive?

F) Given: $\overline{AE} \parallel \overline{BC}$, $m\angle ADC = 87^\circ$,
 Ray \overline{AC} bisects $\angle BAE$
 Ray \overline{CD} bisects $\angle ACB$
 Compute $m\angle ABG$.



**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017 SOLUTION KEY**

Team Round

A) Solution #1: (Squaring both sides of the given equation)

We are looking for x - and y -values for which $\begin{cases} x^2 - y^2 = -17 \\ xy = -72 \end{cases}$.

Concentrating on the second equation, we note $-8 \cdot 9 = -72$ and $(-8)^2 - 9^2 = -17 \Rightarrow \frac{x^2}{y} = \pm \frac{64}{9}$.

[$-9 \cdot 8 = -72$, but $(-9)^2 - 8^2 \neq -17$]

Solution #2: Algebraic

Solving the system $\begin{cases} x^2 - y^2 = -17 \\ 2xy = -144 \end{cases} \Leftrightarrow \begin{cases} x^2 - y^2 = -17 \\ xy = -72 \Rightarrow y = \frac{-72}{x} \end{cases}$, we substitute for y in the

first equation. $x^2 - \left(\frac{-72}{x}\right)^2 = -17 \Leftrightarrow x^4 + 17x^2 - 72^2 = 0$. To get an odd

coefficient for the middle term, the factors of 72^2 must be of opposite parity, so we gather all the 3's and all the 2s, getting $72^2 = 9^2 \cdot 8^2 = 81 \cdot 64$.

The factorization is $(x^2 + 81)(x^2 - 64) = 0 \Rightarrow x = \pm 8, y = \mp 9 \Rightarrow$ the same results.

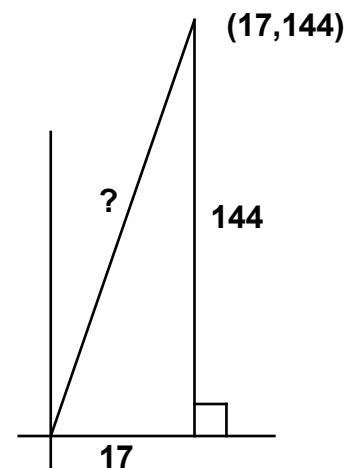
Solution #3: Graphically

$? = 145$, since $145^2 = 17^2 + 144^2$.

Proof: $145^2 - 144^2 = (145 + 144)(145 - 144) = 289(1) = 17^2$

Now these equations are much easier to solve – just ADD.

$$\begin{cases} x^2 - y^2 = -17 \\ x^2 + y^2 = 145 \end{cases} \Rightarrow 2x^2 = 128 \Rightarrow x = \pm 8, y = \pm 9 \Rightarrow \frac{x^2}{y} = \pm \frac{64}{9}$$



B) Of course, we could find the two numbers. Unfortunately, they are complex numbers, i.e., numbers of the form $a + bi$, where $i = \sqrt{-1}$. We would then have to cube each of them, and add these results. Our plan is to avoid all this unpleasantness.

Let the numbers be x and y .

Consider the expansion of $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.

Since $x + y = 1$, we have (re-arranging terms and factoring) $1 = x^3 + y^3 + 3xy(x + y)$.

$$\Leftrightarrow 1 = x^3 + y^3 + 3 \cdot 2(1)$$

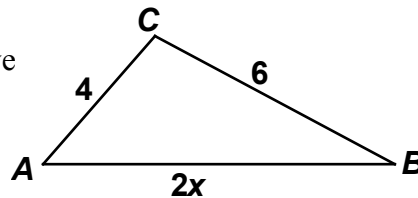
$$\Leftrightarrow x^3 + y^3 = \underline{-5}.$$

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017 SOLUTION KEY**

Team Round

- C) Expecting to use Hero's formula, let $AB = 2x$. This will make computing the semi-perimeter s easy. Using Hero's formula, we have

$$s = \frac{1}{2}(4 + 6 + 2x) = 5 + x.$$



$$\sqrt{(5+x)(5-x)(1+x)(-1+x)} = 3\sqrt{15}$$

$$\Rightarrow (25 - x^2)(-1 + x^2) = 135$$

$$\Rightarrow -25 + 25x^2 + x^2 - x^4 = 135$$

$$\Rightarrow x^4 - 26x^2 + 160 = 0 \Leftrightarrow (x^2 - 10)(x^2 - 16) = 0 \Rightarrow x = 4, \sqrt{10}$$

$$\Rightarrow AB = \underline{\underline{8, 2\sqrt{10}}}.$$

Alternately, area of a triangle is given by $\frac{1}{2}ab\sin C$, where C is the included angle.

$$\text{Thus, the area of } \triangle ABC \text{ is } \frac{1}{2} \cdot 4 \cdot 6 \sin C = 3\sqrt{15} \Rightarrow \sin C = \frac{\sqrt{15}}{4} \Rightarrow \cos C = \pm \frac{1}{4}.$$

Using the Law of Cosines, if

$$AB = c, \quad c^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6 \left(\pm \frac{1}{4} \right) = 52 \pm 12 \Rightarrow AB = \sqrt{64}, \sqrt{40} = \underline{\underline{8, 2\sqrt{10}}}.$$

- D) We must examine the prime factorization of each number.

$$4158000 = 4158(10^3) = 9 \cdot 462(10^3) = 9 \cdot 6 \cdot 77(10^3) = 2^4 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 11$$

$$122850 = 10 \cdot 9 \cdot 1365 = 2 \cdot 3^2 \cdot 5^2 \cdot 273 = 2 \cdot 3^3 \cdot 5^2 \cdot 91 = 2 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 13$$

Common factors are composed of prime factors of 2, 3, 5, and 7, i.e. of the form

$2^a \cdot 3^b \cdot 5^c \cdot 7^d$, where each exponent must be an integer between 0 and the lowest exponent to which that prime occurred in the factorization of each number, inclusive.

Thus, $a : 0 \dots 1$, $b : 0 \dots 3$, $c : 0 \dots 2$, $d : 0 \dots 1$

Thus, there are $2 \cdot 4 \cdot 3 \cdot 2 = \underline{\underline{48}}$ common factors.

**MASSACHUSETTS MATHEMATICS LEAGUE
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Team Round

E) As in the diagram at the right, at least one of the sides of the octagon is parallel to the x -axis, in this case

$$i = 1 \Rightarrow \overline{A_1A_2} \text{ or } i = 5 \Rightarrow \overline{A_5A_6}.$$

The octagon is comprised of 8 isosceles triangles

with vertex angles of $\frac{360}{8} = 45^\circ$ and legs equal to a radius of

the circumscribed circle. Using $\frac{1}{2}ab\sin\theta$ to calculate the

area of a triangle, where θ denotes the included angle, we have $8\left(\frac{1}{2}R^2\sin 45^\circ\right) = 2R^2\sqrt{2} = 50\sqrt{2} \Rightarrow R = 5$ and the

coordinates of the vertices must satisfy $x^2 + y^2 = 25$.

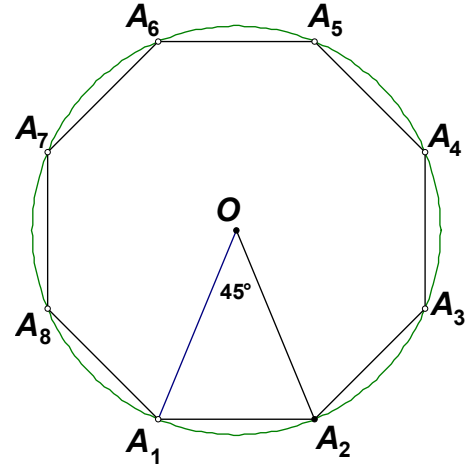
In quadrant I, the only candidates are (3, 4) and

(4, 3). These are not allowed; however, there are two additional candidates in each of the other quadrants

by virtue of symmetry with respect to the x -axis ($x, -y$),

the y -axis ($-x, y$) and the origin ($-x, -y$).

Thus, there are 6 possibilities.



F) Let $m\angle FAD = x^\circ$. Then:

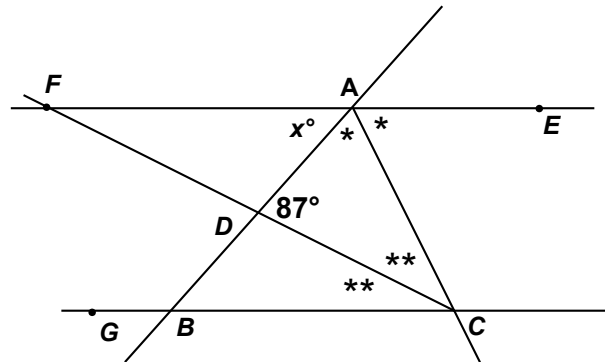
$$m\angle DAE = \frac{1}{2}(180 - x) = 90 - \frac{x}{2}$$

$$m\angle BCA = m\angle CAE = m\angle DAC$$

$$\Rightarrow m\angle DAC = \frac{1}{2}\left(90 - \frac{x}{2}\right) = 45 - \frac{x}{4}.$$

$$\text{Thus, in } \triangle ADC, \left(90 - \frac{x}{2}\right) + \left(45 - \frac{x}{4}\right) + 87 = 180$$

$$\Rightarrow 42 = \frac{3}{4}x \Rightarrow x = 56 \Rightarrow m\angle ABG = \underline{124^\circ}.$$



**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2017 ANSWERS**

Round 1 Algebra 2: Complex Numbers (No Trig)

- A) $\pm 5i\sqrt{3}$ B) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ C) (2, 3)

Round 2 Algebra 1: Anything

- A) 46 B) -96 C) (11, 16.5)

Round 3 Plane Geometry: Area of Rectilinear Figures

- A) 14 B) 12.6 C) 3 : 4

Round 4 Algebra: Factoring and its Applications

- A) $-\frac{3}{5}, \frac{3}{2}$ (or -0.6, 1.5) B) 53 C) $\pm 3, \pm 4$

Round 5 Trig: Functions of Special Angles

- A) 24 B) $\frac{\pi}{6}, \frac{5\pi}{6}$ C) $\frac{5\pi}{48}, \frac{7\pi}{16}, \frac{29\pi}{48}, \frac{15\pi}{16}$

Round 6 Plane Geometry: Angles, Triangles and Parallels

- A) 19 B) (2, 7) C) 138°
Degree symbol not required.

Team Round

- A) $\pm \frac{64}{9}$ D) 48

(both answers required)

- B) -5 E) 6

- C) $8, 2\sqrt{10}$ F) 124